MATH 155 - Chapter 9.4 - Comparison Series:
(Can only be applied to positive series)
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1. **Definition:** Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be positive-term series. We say that $\sum_{n=1}^{\infty} b_n$ **dominates** $\sum_{n=1}^{\infty} a_n$ if $a_n \leq b_n$ for all $n \geq 1$.

2. **Theorem: Direct Comparison Test**

   Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are positive-term series such that $0 < a_n \leq b_n$ for all $n \geq N$ for some positive integer N. (ie. b_n dominates a_n for all $n \geq N$.) Then

   1. If $\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty} a_n$ converges.
   2. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

3. **Theorem: Limit Comparison Test**

   Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are positive-term series, and

   $$\lim_{n\to\infty} \frac{a_n}{b_n} = L.$$

   1. If $0 < L < \infty$, then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge or diverge together.
   2. If $L = 0$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.